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Theory of Bose–Einstein condensation in a microwave-driven interacting magnon gas

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Abstract
Room temperature Bose–Einstein condensation (BEC) of magnons in YIG films under microwave driving has been recently reported. We present a theory for the interacting magnon gas driven out of equilibrium that provides rigorous support for the formation of the BEC. The theory relies on the cooperative mechanisms created by the nonlinear magnetic interactions and explains the spontaneous generation of quantum coherence and magnetic dynamic order when the microwave driving power exceeds a critical value. The results fit very well the experimental data for the intensity and the decay rate of Brillouin light scattering and for the microwave emission from the BEC as a function of driving power.

1. Introduction

A brief account is presented of a theoretical model [1] for the formation of Bose–Einstein condensation (BEC) of magnons at room temperature in films of yttrium–iron garnet (YIG) driven by microwave radiation and its application to interpret the experiments of Demokritov and co-workers [2–6]. BEC occurs when a macroscopic number of bosons occupies the lowest available energy quantum states and has been observed in a few physical systems at very low temperatures [7]. The room temperature experiments of [2–6] have realized earlier proposals for producing BEC of magnons [8] and demonstrated powerful techniques for observing its properties.

The experiments were done at room temperature in epitaxial crystalline YIG films magnetized by in-plane fields. In these films the combined effects of the exchange and magnetic dipolar interactions among the spins produce a dispersion relation (frequency ω_k versus wavevector k) for magnons that has a minimum ω_{k0} at $k_0 \sim 10^5$ cm^{-1}. Magnons are driven parametrically by microwave pulses with frequency f_p and studied by Brillouin light scattering (BLS) [2–4]. The experiments show that, when the microwave power exceeds a threshold value, there is a large increase in the population of the magnons with frequencies in a narrow range around $f_p/2$. Then the energy of the magnons quickly redistributes through modes with lower frequencies down to the minimum frequency $f_{min} = \omega_{k0}/2\pi$ as a result of magnon scattering events. This produces a hot magnon gas that remains decoupled from the lattice for some time due to the long spin–lattice relaxation time.

However, if the microwave power exceeds a second threshold value, much larger than the one for parallel pumping, the magnon population evolves to condense in a narrow region in phase space around the minimum frequency and develops quantum coherence [3, 4]. The coherence of the magnon condensate is also demonstrated by the microwave emission from $k \approx 0$ magnons created by BEC magnon pairs in experiments with the applied field such that the frequency of the $k \approx 0$ magnon is $\omega_0 = 2\omega_{k0}$ [5, 6]. The theoretical model presented here provides rigorous support for the formation of the BEC of magnons in YIG films magnetized in the plane and driven by microwave radiation. The theory relies on cooperative mechanisms made possible by the nonlinear magnetic interactions and explains the spontaneous generation of quantum coherence and magnetic dynamic order in a macroscopic scale when the microwave driving power exceeds a critical value. The results fit very well the experimental BLS and microwave emission data.

2. Microwave excitation of magnons in thin films

The interacting boson gas can be treated with second quantization of the spin excitations with wavevector k and frequency ω_k described by magnon creation and annihilation operators c_k^+ and c_k. The Hamiltonian for the system pumped by a microwave field can be written as

$$H = H_0 + H_{\rm int} + H'(t), \quad (1)$$

where $H_0 = \hbar \sum \omega_k c_k^+ c_k$ is the unperturbed Hamiltonian, $H_{\rm int}$ represents the nonlinear interactions and $H'(t)$ the external microwave driving. The magnetic Hamiltonian is dominated by Zeeman, exchange and magnetic dipolar contributions [9]. As is well known the eigenstates $|n_k\rangle$ of the free Hamiltonian H_0 and of the number operator $n_k = c_k^+ c_k$ can be obtained by applying integral powers of the creation operator to the vacuum. These states have precisely defined numbers of magnons n_k and uncertain phase and are used in nearly all quantum treatments of thermodynamic properties, relaxation mechanisms and other phenomena involving magnons. However, they have zero expectation value for the small-signal transverse magnetization operators m_x and m_y, defined by $\vec{M} = \hat{z}M_z + \hat{x}m_x + \hat{y}m_y$, and therefore do not have a macroscopic wavefunction.

The states that correspond to classical spin waves are the coherent magnon states [10, 11], defined in analogy to the coherent photon states [12]. A coherent magnon state is the eigenket of the circularly polarized magnetization operator $m^+ = m_x + {\rm i}m_y$. It can be written as the direct product of single-mode coherent states, defined as the eigenstates of the annihilation operator, $c_k|\alpha_k\rangle = \alpha_k|\alpha_k\rangle$, where the eigenvalue α_k is a complex number. Although the coherent states are not eigenstates of the unperturbed Hamiltonian and as such do not have a well-defined number of magnons, they have nonzero expectation value for the magnetization $m^+ \propto |\alpha_k|$ and a well-defined phase. The coherent state $|\alpha_k\rangle$ can be expanded in terms of the eigenstates $|n_k\rangle$ and has an expectation value for the number operator $\langle n_k\rangle = |\alpha_k|^2$. The coherent states are not orthogonal to one another, but they form a complete set, so they can be used as a basis for the expansion of an arbitrary state. In order to study the coherence properties of a magnon system, it is convenient to use the density matrix operator ρ and its representation as a statistical mixture of coherent states:

$$\rho = \int P(\alpha_k)|\alpha_k\rangle\langle\alpha_k|\, {\rm d}({\rm Re}\,\alpha_k)\, {\rm d}({\rm Im}\,\alpha_k), \quad (2)$$

where $P(\alpha_k)$ is a probability density. As shown by Glauber [12], if ρ represents a thermal distribution, $P(\alpha_k)$ is a Gaussian function, and if ρ corresponds to a coherent state, $P(\alpha_k)$ is a Dirac δ function.

Spin waves can be nonlinearly excited in a magnetic material by means of several techniques employing microwave radiation, with the microwave magnetic field applied either perpendicular or parallel to the static field. In the parallel pumping process the driving Hamiltonian in equation (1) follows from the Zeeman interaction of the microwave pumping field $\hat{z}h\cos(\omega_{\rm p}t)$ with the magnetic system. One can express the Zeeman interaction in terms of the magnon operators, keeping only terms that conserve energy and show that the driving Hamiltonian for a ferromagnetic film is given by [1]

$$H'(t) = (\hbar/2)\sum_k h\rho_k {\rm e}^{-{\rm i}\omega_{\rm p}t} c_k^+ c_{-k}^+ + {\rm h.c.}, \quad (3)$$

where $\rho_k = \gamma\omega_M[(1 - F_k)\sin^2\theta_k - F_k]/4\omega_k$ represents the coupling of the pumping field h (frequency $\omega_{\rm p}$) with the $\vec{k}, -\vec{k}$ magnon pair with frequency ω_k equal or close to $\omega_{\rm p}/2$, $\omega_M = \gamma 4\pi M$, M is the saturation magnetization, $\gamma = g\mu_{\rm B}/\hbar$ is the gyromagnetic ratio (2.8 GHz kOe^{-1} for YIG), $F_k = (1 - {\rm e}^{-kd})/kd$ is a form factor, d is the film thickness and θ_k is the angle between the wavevector $\vec{k}$ in the plane and the field. For a thick film or bulk sample the coupling is maximum for $\theta_k = 90^\circ$ and vanishes for $\theta_k = 0$. However, in films with kd of the order of 1 or less, F_k is finite and the parallel pumping field can drive waves with any value of θ_k as in the experiments of [2–6].

The Heisenberg equations of motion for the operators c_k and c_k^+ with the Hamiltonian $H = H_0 + H'(t)$ can be easily solved for a pumping field h applied at $t = 0$. The solution shows that, if the field exceeds a critical value $h_{\rm c}$, the number of parametric magnons increases exponentially in time. At large enough values the magnon population saturates due to nonlinear four-magnon interactions that can be represented by the Hamiltonian [9, 13]

$$H^{(4)} = \hbar \sum_{k,k'} (\tfrac{1}{2} S_{kk'} c_k^+ c_{-k}^+ c_{k'} c_{-k'} + T_{kk'} c_k^+ c_{k'}^+ c_k c_{k'}), \quad (4)$$

where the interaction coefficients are determined mainly by the dipolar and exchange energies. For the k values relevant to the experiments the exchange contribution is negligible so that the coefficients in equation (4) are given approximately by $S_{kk'} = 2T_{kk'} = 2\omega_M/NS$, where N is the number of spins S in the sample. Using the Hamiltonian (1) with equations (3) and (4) as the driving and interaction terms one can write the equations for the operators c_k and c_k^+ from which several quantities of interest can be obtained. One of them is the correlation function $\sigma_k = \langle c_k c_{-k}\rangle = n_k {\rm e}^{{\rm i}\varphi_k} {\rm e}^{-{\rm i}2\omega_k t}$ [13], where n_k is the magnon number operator and φ_k is the phase between the states of the pair. It can be shown that, for $h > h_{\rm c}$, the steady-state population of the parametric magnon pairs is [13, 14]

$$\langle n_k\rangle_{ss} = [(h\rho_k)^2 - \eta_k^2]^{1/2} - |\Delta\omega_k|]/2V_{(4)}, \quad (5)$$

where $V_{(4)} = S_{kk} + 2T_{kk} = 4\omega_M/NS$, $\Delta\omega_k = \omega_k - \omega_{\rm p}/2$ is the detuning from the frequency of maximum coupling and η_k is the magnon relaxation rate that was introduced phenomenologically in the equations of motion. It can also be shown that the phase φ_k varies from $-\pi/2$ to π as h increases from $h_{\rm c}$ to infinity. In the range of powers used in [2–6] $\varphi_k \sim -\pi/2$.

Equation (5) shows that magnon pairs with frequency in a narrow range around $\omega_{\rm p}/2$ are pumped by the microwave field when $h > h_{\rm c} = (\eta_k^2 + \Delta\omega_k^2)^{1/2}/\rho_k$. Note that, for $\omega_k = \omega_{\rm p}/2$, $h_{\rm c} = \eta_k/\rho_k$. In the reported experiments the minimum $h_{\rm c}$ corresponds to a critical power $p_{\rm c}$ in the range of 100 μW–1 mW determined by the experimental geometry and the spin–lattice relaxation rate in YIG, $\eta_{\rm SL} \sim 2 \times 10^6$ s^{-1} [3]. However, when very short microwave pulses are used, much higher power levels are required to reduce the rise time and to build up large magnon populations. In this case the relaxation rate that prevails in the dynamics is dominated by magnon–magnon scattering $\eta_{\rm m} \sim 25\eta_{\rm SL} = 5 \times 10^7$ s^{-1} [1], so one can define a critical field $h_{c1} = \eta_m/\rho_k = h_c\eta_m/\eta_{SL}$ for driving magnons out of equilibrium from the magnon heat bath. Since the driving power p is proportional to h^2, one can write from equation (5) an expression for the steady-state total number of parametric magnons with $\omega_k \approx \omega_p/2$:

$$N_p = r_p n_H[(p - p_{c1})/p_{c1}]^{1/2}, \quad (6)$$

where $n_H \equiv \eta_m/2V_{(4)} = \eta_m NS/8\omega_M$ and r_p is a factor that represents the number of pumped modes weighted by a factor relative to the number of magnons of the mode with maximum coupling.

3. The model for the dynamics of BEC in the microwave-driven interacting magnons

In the experiments of [2–6] magnon pairs are parametrically driven by parallel pumping in a YIG film at large numbers compared to the thermal values. The population of these magnons is quickly redistributed over a broad frequency range down to the minimum frequency due to four-magnon scattering events which conserve the total number of magnons. Since the spin–lattice relaxation time in YIG is much longer than the intermagnon decay time a quasi-equilibrium hot magnon gas is formed that remains decoupled from the lattice for several hundred ns with an essentially constant number of magnons. Thus, assuming that the system is in quasi-equilibrium and neglecting the interaction between magnons, the occupation number of the state with energy $\hbar\omega$ at temperature T is given by the BE distribution

$$n_{BE}(\omega, \mu, T) = 1/\{\exp[(\hbar\omega - \mu)/k_B T] - 1\}, \quad (7)$$

where μ is the chemical potential. Considering that the number of magnons N_p pumped into the system by the microwave driving is much larger than the thermal number in the frequency range from ω_{k0} to $\omega_p/2$, we can write $N_p = \int D(\omega)n_{BE}(\omega, \mu, T)\,d\omega$, where $D(\omega)$ is the magnon density of states and the integral is carried out over the range $\omega_p/2 - \omega_{k0}$. As the microwave power is raised, the total number of magnons increases and the chemical potential rises. At a high enough power, μ reaches the energy corresponding to ω_{k0}, resulting in an overpopulation of magnons with that frequency so that the gas is spontaneously divided in two parts, one with the magnons distributed according to equation (7) and another with the magnons accumulated in states near the minimum energy.

While the thermodynamic interpretation of the experiments in [2, 3] is satisfactory and explains qualitatively several observed features, it fails in providing quantitative results to compare with data and does not explain the observed emergence of quantum coherence in the BEC. Here we show that the cooperative action of the magnon gas through the four-magnon interaction can provide the mechanism for the onset of quantum coherence in the BEC. The theory relies in part on some assumptions based on the experimental observations and on some approximations to allow an analytical treatment of the problem. They are ultimately justified by the good agreement between theory and experimental data.

We assume that, after the hot magnon reservoir is formed by the redistribution of the primary magnons, the correlation between the phases of the magnon pairs lasts for a time that can be as large as $4/\eta_m$, which is about 100 ns in the experiments of [2–6]. This is sufficient time for the four-magnon interaction to come into play for establishing a cooperative phenomenon to drive a specific k mode. The effective driving Hamiltonian for this process is obtained from equation (4) by taking averages of pairs of destruction operators of reservoir magnons to form correlation functions:

$$H'(t) = \hbar \sum_{k_R} \tfrac{1}{2} S_{kk_R} n_{k_R} e^{i\varphi_{k_R}} e^{-i2\omega_{k_R} t} c_k^+ c_{-k}^+ + \text{h.c.} \quad (8)$$

Equation (8) has a form similar to the Hamiltonian (4) for parallel pumping, revealing that under appropriate conditions magnon pairs can be pumped out of equilibrium in the gas. Consider that the population of the primary magnons is distributed among the N_R modes k_R in the magnon reservoir, so that with equation (6) we can write an expression for the average population of the modes as a function of power p:

$$n_R = rn_H[(p - p_{c1})/p_{c1}]^{1/2}, \quad (9)$$

where $r = r_p/N_R$. The number of magnons in each state k_R can be written approximately as $n_{k_R} = f_{BE}(\omega_{kR})n_R$, where $f_{BE}(\omega) = n_{BE}(\omega)/C_{BE}$, $C_{BE} = [\int n_{BE}\,d\omega]/\Delta\omega_R$ is a normalization constant and $\Delta\omega_R = \omega_p/2 - \omega_{k0}$. So the relevant quantity for determining the frequency dependence of the coefficient in equation (8) is the spectral density $G(\omega) = D(\omega)f_{BE}(\omega)$ which has a peak at ω_{k0} that becomes sharper as the chemical potential rises and approaches the minimum energy. Thus, as the microwave pumping power increases and $(\hbar\omega_{k0} - \mu)/k_B T$ becomes very small the peak in $G(\omega)$ dominates the coefficient in equation (8). This establishes a cooperative action of the modes with ω_{k_R} close to ω_{k0} that can drive magnon pairs, similarly to the parallel pumping process. Considering that the pumping is effective for frequencies ω_{k_R} in the range $\omega_{k0} \pm \eta_m$, one can write an effective Hamiltonian for driving k_0, $-k_0$ magnon pairs as

$$H'_{eff}(t) \cong \hbar(h\rho)_{eff}\, e^{-i2\omega_{k0}t} c_{k_0}^+ c_{-k_0}^+ + \text{h.c.}, \quad (10)$$

where $(h\rho)_{eff} = -iG(\omega_{k0})\eta_m V_{(4)} n_R/2$ represents an effective driving field and the factor $-i$ arises from the phase between pairs that is approximately $-\pi/2$ in the range of power of [2–6]. From the analysis in section 2 one can see that there is a critical number of reservoir modes above which they act cooperatively to drive the k_0, $-k_0$ magnons. Since equation (10) has the same form as (3), the population of the k_0 mode driven by the effective field and saturated by the four-magnon interaction is calculated in the same manner as for the direct parallel pumping process. From equation (5) with $\Delta\omega_k = 0$ we have

$$n_{k0} = [|(h\rho)_{eff}|^2 - \eta_m^2]^{1/2}/2V_{(4)}. \quad (11)$$

With equations (9) and (11) one can write the population of the k_0 mode in terms of the power p

$$n_{k_0} = n_H[(p - p_{c2})/(p_{c2} - p_{c1})]^{1/2}, \quad (12)$$

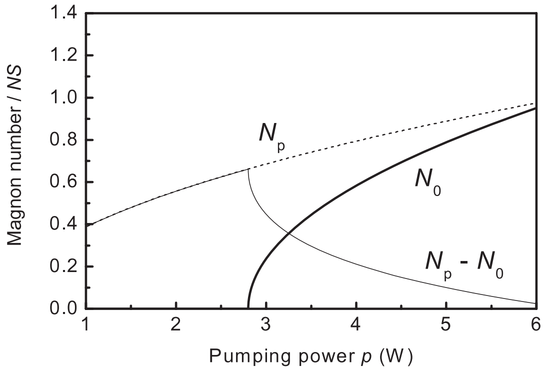

Figure 1. Variation with microwave pumping power of the normalized number of primary pumped magnons N_p, of the BEC population N_0 and of the uncondensed magnons $N_p - N_0$.

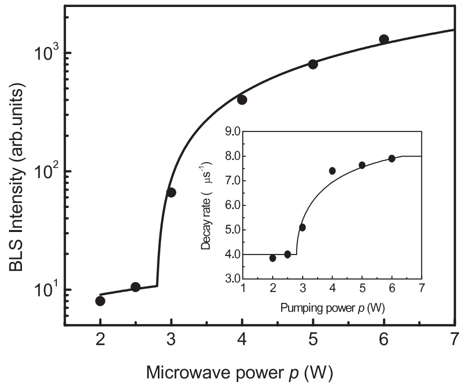

Figure 2. Fit of the theoretical result (solid line) to the experimental data (symbols) of Demokritov and co-workers [4] for the BLS peak intensity at $f_{\min}$ as a function of microwave pumping power. The inset shows a fit of theory (solid line) to data (symbols) for the decay rate of the BLS peak at $f_{\min}$ as a function of microwave pumping power.

where $p_{c2} = p_{c1}\{1 + 16/[r\eta_m G(\omega_{k0})]^2\}$ is the critical power for the formation of the BEC and is much larger than p_{c1}. Equation (12) reveals that for $p \geqslant p_{c2}$ the k_0 magnons are pumped-up out of equilibrium as a result of a spontaneous cooperative action of the reservoir modes and, as will be shown in the next section, they are in a coherent magnon state.

4. Quantum coherence of the Bose–Einstein magnon condensate

In order to study the coherence properties of the k_0 mode pumped above threshold one has to use methods of statistical mechanics appropriate for boson systems interacting with a heat bath [1, 14]. Using the representation of coherent states with eigenvalue $\alpha_k = a_k \exp(i\phi_k)$ one can show that the probability density $P(\alpha_k)$ in equation (2) obeys a Fokker–Planck equation that has a stationary solution:

$$P(x) = C \exp(\tfrac{1}{2}Ax^2 - \tfrac{1}{6}x^6), \qquad (13)$$

where C is a normalization constant that makes the integral of the probability density $P(x)$ equal to unity, $x = (2/n_H^2 \bar{n}_{k0})^{1/6} a_k$ represents a normalized magnon amplitude and the parameter A is given by

$$A = (2/n_H^2 \bar{n}_{k0})^{2/3}[|(h\rho)_{\text{eff}}|^2 - \eta_m^2]^{1/2}/2V_{(4)}$$
$$\equiv (2/n_H^2 \bar{n}_{k0})^{2/3} n_{k_0}^2. \qquad (14)$$

From equation (14) we see that, for $p < p_{c2}$, the parameter A is negative. In this case $P(x)$ given by equation (14) behaves as a Gaussian distribution, typical of systems in thermal equilibrium with incoherent magnon states. On the other hand, for $p > p_{c2}$, $A > 0$ and $P(x)$ consists of two components, a coherent one convoluted with a much smaller fluctuation with Gaussian distribution. Since $P(x)$ has a variance proportional to A^{-1}, for $A \gg 1$ it becomes a delta-like function, characteristic of coherent magnon states [12]. Note that $P(x)$ has a peak at $x_0 = A^{1/4}$, so that it represents a coherent state with an average number of magnons given by $x_0^2 = A^{1/2}$. From equation (14) we see that this corresponds to a magnon number a_0^2 which is precisely the value n_{k_0} given by equation (11). This means that the magnons ω_{k0} driven cooperatively by the reservoir modes are in quantum coherent states that have, from equation (12), a small-signal magnetization $m^+ \propto (p - p_{c2})^{1/4}$. Thus m^+ is the order parameter of the BEC.

The calculations presented so far are valid for magnon pairs with frequencies and wavevectors in the vicinity of ω_{k0} and $k_0, -k_0$. The dynamics for several modes can be treated approximately assuming that the condensate consists of p_{k0} modes all governed by the single-mode equations. As will be shown later the experimental data are well fitted by this model with $p_{k0} = 4.4 \times 10^3$ which is a very small number compared to the number of reservoir states $N_R \sim 10^9$. Thus we can write the number of magnons in the condensate as $N_0 = p_{k0} n_{k0}$.

Figure 1 shows the variation with microwave driving power of the total number of magnons pumped into the system N_p given by equation (6) and the population of the BEC magnons N_0 calculated with equation (12) using parameters obtained from a fit of theory to data, $p_{k0} = 4.4 \times 10^3$ and $r_p = 5.0 \times 10^2$. Also shown is the number of uncondensed magnons $N_p - N_0$ as a function of p. Clearly the number of condensate magnons approaches the total number of particles while the number of uncondensed magnons vanishes as the power increases above the critical value. This is also a typical feature of a BEC.

5. Comparison to experimental data

5.1. Brillouin light scattering

In the experiments of [3] the coherence properties of the excited magnon states emerge clearly in the behaviour of the intensity of the BLS peak at $f_{\min}$. As argued in [3], for incoherent scatterers the BLS intensity is proportional to their number, whereas for coherent scatterers it is proportional to the number squared. To compare theory with data we express the BLS intensity in terms of the microwave power p in two regimes, $p < p_{c2}$ and $p \geqslant p_{c2}$. Figure 2 shows a fit to data using $I^{\text{inc}} = c_1(p - p_{c1})^{1/2}$ and $I^{\text{coh}} = c_2(p - p_{c2})$, with $c_1 = 6.7$, $c_2 = 370.0$ and $p_{c2} = 2.8$ W. As shown in [1] the parameters are consistent with the values $r_p = 5 \times 10^2$ and $p_{k0} = 4.4 \times 10^3$ used to fit other experimental data.

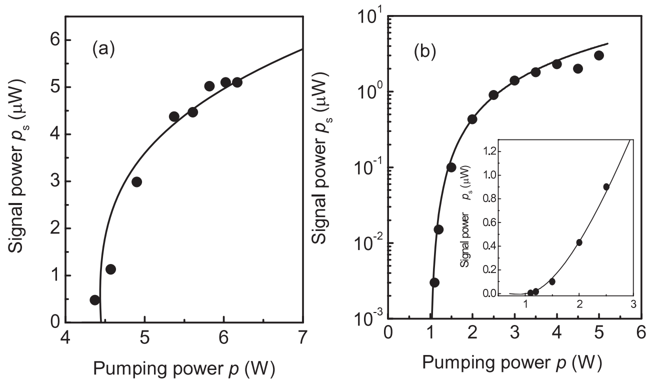

Figure 3. (a) Microwave emission signal power (μW) versus pumping power (W). Symbols represent the experimental data of [5] and the solid line is the fit with theory; (b) microwave emission signal power versus pumping power. Symbols represent the experimental data of [6] and the solid line is the fit with theory. The inset shows the parabolic dependence on the power near the critical value observed experimentally.

The BLS experiments with short microwave pulses also provide data on the variation with p of the time decay of the BLS peak at $f_{\min}$ due to the relaxation of the magnons to the lattice. As shown in [3] as the microwave power increases above the critical value $p_{c2} = 2.8$ W the decay rate doubles in a stepwise manner. This fact was interpreted as an indication of the emergence of coherence of the magnons in the condensed state. The argument is that, if the n_k magnons causing the scattering are incoherent, the intensity of the BLS peak falls exponentially in time with the same rate of the magnons. However, if the magnons are coherent the BLS intensity should follow n_k^2 so that its decay rate is twice that of the magnons. As shown in figure 1 the number N_0 of coherent magnons in the BEC increases with increasing microwave power above the threshold and approaches the total number N_p of magnons pumped in the system at $p \sim 6$ W. At any given power level the difference $N_p - N_0$ represents the number of incoherent magnons concentrated in a narrow range of frequencies around $f_{\min}$, thus contributing to the BLS intensity [1]. Assuming that the decay rate η_{BLS} of the BLS peak is a linear combination of the rates for incoherent and coherent scatterers we can write $\eta_{BLS} = 2\eta_{SL}[(N_p - N_0)/N_p] + 4\eta_{SL}N_0/N_p$, where η_{SL} is the spin–lattice relaxation rate of the magnetization and $2\eta_{SL} \approx 4 \times 10^6$ s^{-1} is the relaxation rate of the magnon number. The solid line in the inset of figure 3 represents the theoretical fit to data [3] using $p_{k0} = 4.4 \times 10^3$ and $r_p = 5.0 \times 10^2$.

5.2. Microwave emission

As observed in [5, 6] if the field applied to the YIG film is such that the frequency of the $k \approx 0$ magnon is $\omega_0 = 2\omega_{k0}$, a microwave signal with frequency ω_0 is emitted by $k \approx 0$ magnons created by pairs of BEC magnons $k_0, -k_0$ through a three-magnon confluent process. As shown in [1, 15] using the total Hamiltonian including driving and three- and four-magnon interactions, one can obtain the equations of motion for the magnon operators c_0 and c_{k0}. The equations are solved numerically to give the steady-state magnon populations n_0 and n_{k0} as a function of pumping power. The total average power radiated by the uniform magnetization precessing about the static field with frequency ω_0 is given by [1, 15] $\langle P\rangle \cong (V^2\omega_0^4 M^2/c^3)n_0'$, where V is the volume of the emission region, c is the speed of light and $n_0' = n_0/NS$. Figure 3 shows fits of the expression $p_s = Cn_0'$ to data. In (a) the symbols represent the data of [5] and the solid line represents the fit with using $C = 14.3$ μW, $p_{k0} = 4.4 \times 10^3$ and $p_{c2} = 4.45$ W. In (b) the symbols are the data of [6] using an incoherent pumping source and the fitting parameters are $C = 1.7 \times 10^4 \mu$W, $p_{k0} = 5.0$ and $p_{c2} = 1.0$ W. The model reveals the contrasting behaviour of the BEC when the pumping changes from coherent to incoherent. The low value of p_{k0} used to fit the data of [6] indicates that with incoherent pumping the number of magnons in the system is significantly smaller than with coherent pumping, thus reducing the number of states occupied by the BEC. Again the fit of theory to data is very good.

6. Summary

We have presented a theoretical model for the dynamics of magnons in a YIG film driven by microwave radiation far out of equilibrium that provides rigorous support for the formation of Bose–Einstein condensation of magnons in the experiments of Demokritov and co-workers [2–6]. The model relies on the cooperative action of magnons with frequencies close to the minimum of the dispersion relation through the nonlinear four-magnon interactions. The theory provides the basic requirements for the characterization of a BEC, namely: (a) the onset of the BEC is characterized by a phase transition that takes place as the microwave power p is increased and exceeds a critical value p_{c2}; (b) the magnons in the condensate are in coherent states and, as such, they have nonzero small-signal transverse magnetization that is the order parameter of the BEC; (c) for $p > p_{c2}$ the magnon system separates into two parts, one in thermal equilibrium with the reservoir and one with N_0 coherent magnons having frequencies and wavevectors in a very narrow region of phase space. As the microwave power increases further N_0 approaches the total number of magnons pumped into the system characterizing unequivocally a Bose–Einstein condensation. The results of the model fit quite well the experimental data obtained with Brillouin light scattering and with microwave emission with consistent values for the various material and fitting parameters. We note that the theory also applies to the BEC of magnons in superfluid ^{3}He [16].

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